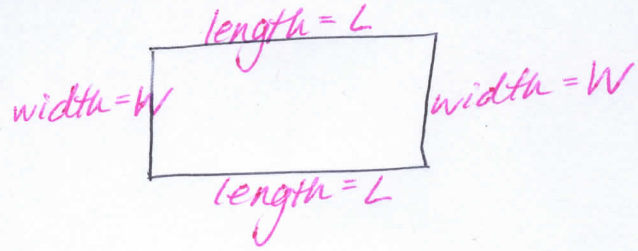
# Section [redacted] 3.7

1) A campground owner has 800 meters of fencing. He wants to enclose a rectangular field. Let w represent the width of the field. Follow these steps to find the dimensions of the field that yields the largest area.

sketch a diagram




a) write an expression for the length of the field

perimeter of region to be fenced is given by

$$P = 2L + 2W$$

$$800 = 2L + 2W$$

solve this for L to answer question

$$800 = 2L + 2W$$
$$-2W \qquad -2W$$

$$\frac{800}{2} - \frac{2W}{2} = \frac{2L}{2}$$

$\boxed{\text{ANSWER: } 400 - W = L}$

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b) write an equation for the area of the field

area of region $= A = LW$

replace L with $(400-W)$

$\boxed{\text{ANSWER: } A = (400-W)W}$

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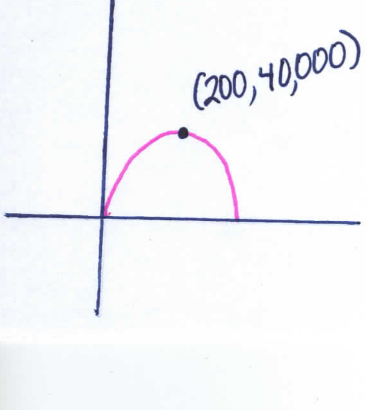
c) find the value of w leading to the maximum area

graph $y_1 = (400-x)(x)$




X min 0
X max 400
y min 0
y max 100,000

find maximum point

(200, 40000)

x-coordinate of vertex gives optimal width

$\boxed{\text{ANSWER: 200 meters wide}}$

d) find the value of L leading to the maximum area

find optimal length

$L = 400 - W$

$= 400 - (200)$ substitute optimal width for W

$= 200$

$\boxed{\text{ANSWER: } 200 \text{ meters long}}$

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e) find the maximum area

y-coordinate of vertex gives maximum area

(200, 40000)

$\boxed{\text{ANSWER: } \quad Area = 40000\, m^2}$

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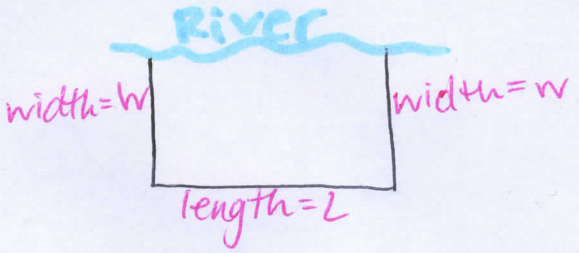
3) A campground owner has 1400 meters of fencing. He wants to enclose a rectangular field bordering a river, with no fencing needed along the river, and let W represent the width of the field. Follow these steps to find the dimensions of the field that yields the most area.




$P = L + 2W$

a) write an expression for the length of the field

replace P with 1400

$1400 = L + 2w$

Solve for L

$\boxed{1400 - 2w = L}$

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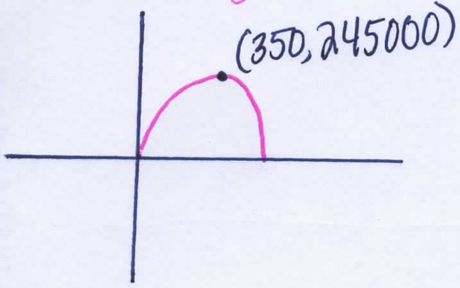
b) write an equation for the area of the field

$A = LW$

$\boxed{A = (1400 - 2w)w}$

c) find the value of W leading to the maximum area

sketch the graph $y = (1400-2x)(x)$




Xmin 0
Xmax 700
y min 0
X max 250,000

value of W that leads to maximum area

$\boxed{\text{Width} = 350 \text{ meters}}$

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d) find the value of L leading to the maximum area

$L = 1400 - 2W$

replace $W = 350$

$L = 1400 - 2(350)$

$L = 1400 - 700$

$L = 700$

Value of L that leads to maximum area

$\boxed{\text{length} = 700 \text{ meters}}$

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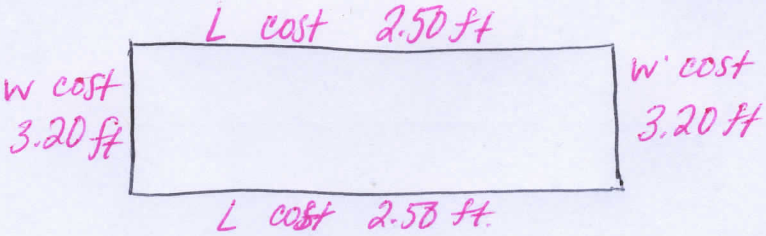
e) find maximum area

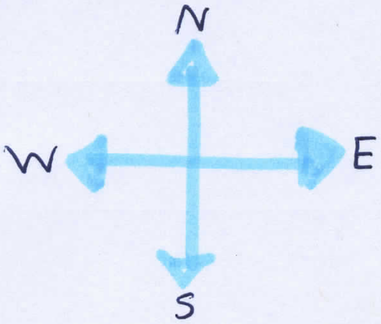
y-coordinate of vertex is maximum area

$(350, 245000)$

$\boxed{\text{maximum area that can be enclosed} = 245{,}000 \text{ square miles}}$

5) A fence must be built to enclose a rectangular area of 20,000 square feet. Fencing material costs \$2.50 per foot for the two sides facing north and south (call these sides the length, and \$3.20 per foot for the other two sides (call these sides the width). Follow these steps to find the cost of the least expensive fence.




a) write an equation for the length of the field

$A = LW$ I create an area formula because I know the desired area

$$\frac{20{,}000}{w} = \frac{LW}{w}$$

$$\boxed{\frac{20{,}000}{w} = L}$$

b) write an equation for the cost of the field

Cost = 2.50(total lengths) + 3.20(total widths)

$= 2.50(2L) + 3.20(2w)$

$$\boxed{C = 5L + 6.40w}$$

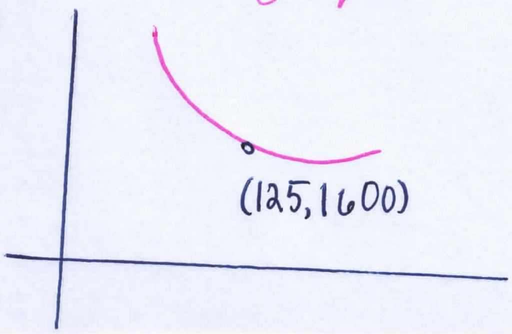
c) find the value of w leading to the minimum cost

$C = 5L + 6.40w$

replace $L$ with $\frac{20000}{w}$

$C = 5\left(\frac{20000}{w}\right) + 6.40w$

graph




X min 0

X max 200 $\sqrt{\text{area}}$

y min 0

y max 3000

x-coordinate of vertex gives optimal width

value of w that gives lowest cost

$$\boxed{W = 125 \text{ ft.}}$$

d) find the value of L leading to the minimum cost

$$L = \frac{20000}{W}$$

$$L = \frac{20000}{125}$$ substitute optimal width (125) in for W

$L = 160$ value of L that gives lowest cost

$\boxed{\text{length} = 160 \text{ ft.}}$

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e) find minimum cost

lowest cost is y-coordinate of vertex

$\boxed{\text{cost} = \$1,600}$

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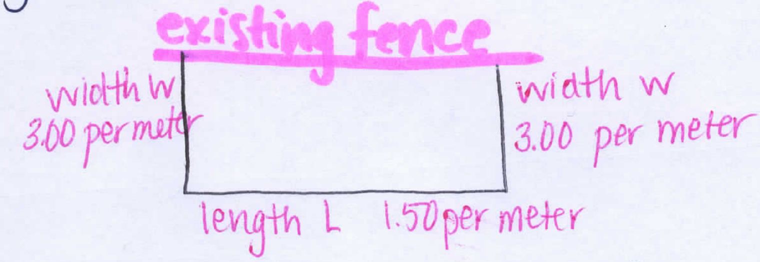
7) A fence must be built in a large field to enclose a rectangular area of 25,000 square meters. One side of the area is bounded by an existing fence; no fence is needed there. Material for the fence costs \$3.00 per meter for the two ends, and \$1.50 per meter for the side opposite the existing fence. Find the cost of the least expensive fence.




a) write an equation for the length of the field

$$A = LW$$

$$\frac{25,600}{W} = \frac{LW}{W}$$ solve for L

$$\boxed{\frac{25,600}{w} = L}$$

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b) write an equation for the cost of the field

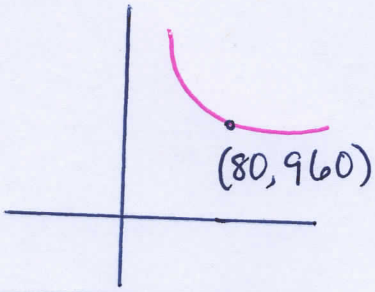
$$\text{cost} = 1.50L + 3.00(2w)$$

$$= 1.50L + 6.00w$$

$$\boxed{C = 1.50\left(\frac{25600}{w}\right) + 6.00w}$$

c) find the value of W leading to the minimum cost

sketch graph, find minimum




X min 0

X max 160

y min 0

y max 3000

value of W that leads to lowest cost $\boxed{W = 80 \text{ meters}}$

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d) find the value of L leading to the minimum cost

$$L = \frac{25600}{W}$$

$$= \frac{25600}{80}$$ replace with optimal width

$\boxed{L = 320 \text{ meters}}$

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e) find the minimum cost

y-coordinate of vertex

(80, 960)

$\boxed{\$960 = \text{lowest cost}}$

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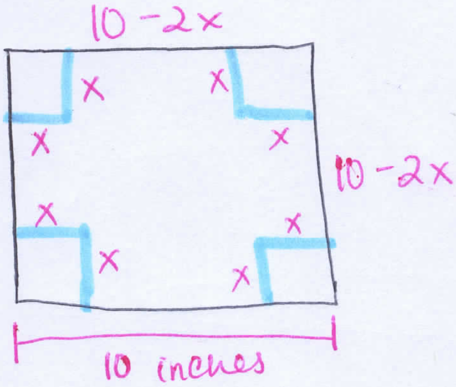
9) An open box with a square base is to be made from a square piece of cardboard 10 inches on a side by cutting out a square (x inches by x inches) from each corner and turning up the sides. (round to 2 decimals if needed)

a)




completed box:

length = 10-2x

width = 10-2x

heigth = X

b) write an equation for the volume of the box

$V = Lwh$

$= (10-2x)(10-2x)(x)$

$= (10-2x)^2(x)$

$\boxed{V = x(10-2x)^2}$

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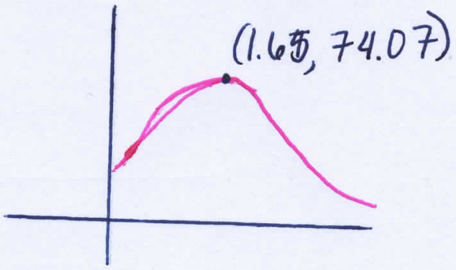
c) graph the volume function using your graphing calculator and find the value of x that makes v the largest

x-coordinate

X min 0
X max 5
y min 0
y max 100+




cutting out a 1.65 inch square gives maximum volume

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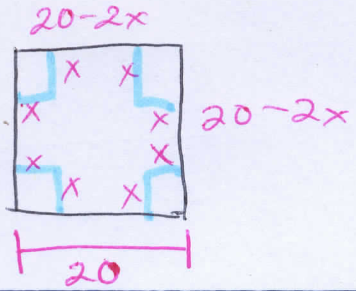
11) An open box is to be made by cutting a square corner of a 20 inch by 20 inch piece of metal then folding up the sides. What size square should be cut from each corner to maximize volume? (round to 2 decimals if needed)

a)




completed:

length = 20-2x
width = 20-2x
heigth = x
volume = Lwh

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b) write an equation for the volume of the box

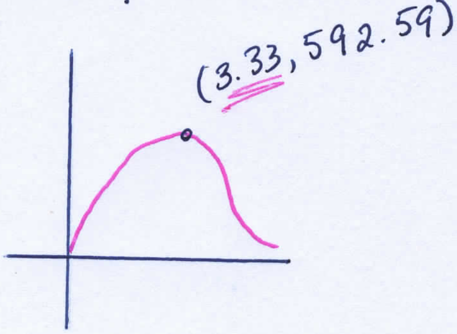
$V = Lwh$

$= (20-2x)(20-2x)(x)$

$\boxed{V = x(20-2x)^2}$

c) graph the volume function using your graphing calculator and find the value of x that makes V the largest (round to 2 decimal places if needed)

xmin 0
x max 10
y min 0
y max 1000




cutting a 3.33 inch square will maximize volume